Using Duration to Approximate Changes in Price (Present Value)

Recall the following formulas from Module 4:

Macaulay Duration:

$$MacD = \frac{\sum t \cdot R_t \cdot v^t}{\sum R_t \cdot v^t}$$

Modified Duration:

$$ModD = -\frac{P'(i)}{P(i)} = v \cdot MacD$$

$P(i)$ is the present value, or price, as a function of *i*.

We can use these durations to approximate the change in price (present value) for a given change in interest rates. Namely, if the interest rate changes from i_{old} to i_{new}, then

1. The **first-order modified approximation** for the change in price is

   $\Delta P \approx -P \cdot ModD \cdot \Delta i$, where P and $ModD$ use i_{old}, and $\Delta i = i_{new} - i_{old}$ is the change in the interest rate.

2. The **first-order Macaulay approximation** for the change in price is

   $\Delta P \approx P \cdot \left[\left(\frac{1+i_{old}}{1+i_{new}}\right)^{MacD} - 1\right]$, where P and $MacD$ use i_{old}.

Example: A bond has a price of 7025 using an annual effective yield rate of 7%. Using the same yield rate, the Macaulay duration of the bond 4.946 years.

(a) Using the first-order modified approximation, calculate the price of this bond if the yield rate is change to 6.5% annual effective.
(b) Using the first-order Macaulay approximation, calculate the price of this bond if the yield rate is change to 6.5% annual effective.
(c) Using the first-order modified approximation, calculate the price of this bond if the yield rate is change to 7.2% annual effective.
(d) Using the first-order Macaulay approximation, calculate the price of this bond if the yield rate is change to 7.2% annual effective.

Solution:

(a) $\Delta P \approx -P \cdot ModD \cdot \Delta i$, where $P = 7025$, $ModD = \frac{4.946}{1.07}$, and $\Delta i = -0.005$

Therefore, $\Delta P \approx -(7025) \cdot \left(\frac{4.946}{1.07}\right) \cdot (-0.005) = 162.362 \cdots$, and so the new price is approximately $P + \Delta P \approx 7025 + 162 = 7187$.

(b) $\Delta P \approx P \cdot \left[\left(\frac{1+i_{old}}{1+i_{new}}\right)^{MacD} - 1\right]$, where $P = 7025$ and $MacD = 4.946$.

Therefore, $\Delta P \approx (7025) \cdot \left[\left(\frac{1.07}{1.065}\right)^{4.946} - 1\right] = 164.643 \cdots$, and so the new price is approximately $P + \Delta P \approx 7025 + 165 = 7190$.

(c) $\Delta P \approx -P \cdot ModD \cdot \Delta i$, where $P = 7025$, $ModD = \frac{4.946}{1.07}$, and $\Delta i = 0.002$

Therefore, $\Delta P \approx -(7025) \cdot \left(\frac{4.946}{1.07}\right) \cdot (0.002) = -64.945 \cdots$, and so the new price is approximately $P + \Delta P \approx 7025 - 65 = 6960$.

(d) $\Delta P \approx P \cdot \left[\left(\frac{1+i_{old}}{1+i_{new}}\right)^{MacD} - 1\right]$, where $P = 7025$ and $MacD = 4.946$.

Therefore, $\Delta P \approx (7025) \cdot \left[\left(\frac{1.07}{1.072}\right)^{4.946} - 1\right] = -64.585 \cdots$, and so the new price is approximately $P + \Delta P \approx 7025 - 65 = 6960$.